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Incremental Shifts in Classroom Practice

Supporting Implementation of the Common Core State Standards–Mathematics

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WestEd's Evaluation of the Math in Common Initiative

Math in Common® is a five-year initiative, funded by the S. D. Bechtel, Jr. Foundation, that supports a formal network of 10 California school districts as they are implementing the Common Core State Standards in Mathematics (CCSS-M) across grades K–8. Math in Common grants have been awarded to the school districts of Dinuba, Elk Grove, Garden Grove, Long Beach, Oakland, Oceanside, Sacramento City, San Francisco, Sanger, and Santa Ana.

WestEd is providing developmental evaluation services over the course of the initiative. The evaluation plan is designed principally to provide relevant and timely information to help each of the Math in Common districts meet their implementation objectives. The overall evaluation centers around four central themes, which attempt to capture the major areas of work and focus in the districts as well as the primary indicators of change and growth. These themes are:

» Shifts in teachers' instructional approaches related to the CCSS-M in grades K–8.

» Changes in students' proficiency in mathematics, measured against the CCSS-M.

» Change-management processes at the school district level, including district leadership, organizational design, and management systems that specifically support and/or maintain investments in CCSS-M implementation.

» Development and sustainability of the Math in Common Community of Practice.

Together, the Math in Common districts are part of a community of practice in which they share their progress and successes, as well as their challenges and lessons learned about supports needed for CCSS-M implementation. Learning for district representatives is supported by WestEd team members who provide technical assistance related to goal-setting and gathering evidence of implementation progress (e.g., by advising on data collection instruments, conducting independent data analyses, and participating in team meetings to support leadership reflection). An additional organizational partner, California Education Partners, works with the community of practice by offering time, tools, and expertise for education leaders to work together to advance student success in mathematics. California Education Partners organizes Leadership Convenings three times per year, summer Principal Institutes, "opt-in" conferences on high-interest topics (e.g., formative assessment), and cross-district visitation opportunities.

Executive Summary

Teaching mathematics is complex work. Effectively implementing the Common Core State Standards-Mathematics (CCSS-M) requires teachers to engage students in meaningful learning in which students make sense of mathematical ideas and representations, and communicate and reason mathematically. Teachers must also ensure that they are providing mathematical access to all of their students. Instead of expecting teachers to implement the large-scale changes called for in the CCSS-M overnight, change may be more likely and more sustainable if teachers are encouraged to shift their practice incrementally in a continuous improvement model (Star, 2016; Hiebert & Morris, 2012; Stigler & Hiebert, 2004).

Accordingly, the expectation should be for small yet powerful changes that teachers can implement relatively easily in their instruction (Star, 2016). For example, teachers may initially implement manageable new ideas that make sense to them, such as:

- Math talks to support students to conceptualize and represent operations
- Structures and practices to support student-to-student discourse in small group work
- Counting objects to support students to sort, organize, and count by groups
- Choral counting to engage students in reasoning, predicting, looking for patterns, and justifying things they notice in their counting

Incorporating any of the above changes can make small yet powerful differences in a classroom (Star, 2016), but it is the accumulation of these types of incremental shifts over time that will most likely result in the fullest implementation of the CCSS-M.

Math in Common classroom observation study

As part of its evaluation of the Math in Common (MiC) initiative, WestEd sought to document shifts in teachers' instructional approaches related to the CCSS-M in grades K–8. In order to gather data on classroom practice, pairs of WestEd research staff visited 141 elementary and middle school classrooms in eight California school districts during the 2015/16, 2016/17, and 2017/18 academic years to observe and analyze mathematics lessons. Districts chose teachers each year based on grade level, availability, and interest. We visited some classrooms just once and others multiple times throughout the years, depending on teachers' and districts' interest and availability.

In order to better understand the nature of incremental shifts in teachers' classroom practice as they continued their implementation of CCSS-M, WestEd secured permission from four districts to have the same 16 teachers who had been observed twice in 2016/17 be observed again twice in 2017/18, for a total of four observations per teacher over two years.

In general, the observed teachers showed shifts in practice in differing amounts and categories, based on our common observation rubric. After analyzing the 16 teacher ratings and observation notes across the two years, five key themes emerged describing the teachers' exhibited shifts in classroom practice:

- Representing and linking mathematical ideas
- Student access to mathematical content
- Teachers' mathematical content knowledge
- Collaboration supports teachers' identity
- Continuing to learn and change

Recommendations for supporting teachers' implementation of the CCSS-M

Based on our focused study of teachers' classroom practice over two years, we offer the following set of recommendations to other districts who seek to build support for teachers as they work to implement the CCSS-M:

- **Support teachers in making continuous incremental changes**. All of the cases illustrate the importance of honoring teachers as they take on the monumental task of implementing the CCSS-M. Small but powerful changes that teachers can incorporate fairly easily in their practice can allow teachers to try out new ideas without feeling as though they need to totally overhaul their teaching. These changes can be viewed as modifications to what they are already doing, which increase the likelihood of the changes sticking. Examples of small changes include number talks, counting collections, problems of the week, choral counting, or small-group student discourse.
- **Develop structures and expertise to support increased mathematical content knowledge.** As one of the teacher cases highlighted, teachers who have multiple levels of support for their pedagogical content knowledge gain more flexible procedural and conceptual mathematics knowledge that helps them implement the CCSS-M. Types of support that may help teachers continuously improve their mathematics teaching include site-based lesson design collaboration, knowledgeable principal support, content-focused professional development, content coaching and mentoring, lesson study, and cross-district grade-level meetings.
- **Support teachers in providing all students with access to mathematical content.** In order for teachers to implement the CCSS-M effectively for all students, classroom activity structures should invite and support the active mathematical engagement of all of the students in the classroom in a meaningful way. Classrooms in which a small proportion of students get most of the attention are not equitable, no matter how rich the content. Teachers need support to understand how to actively support broad and meaningful mathematical participation, and how and when to use successful participation structures. In addition, teachers need to understand what it means in practice for students to actively share and discuss their ideas. Focusing on discourse strategies can help students learn to explain their ideas and reasoning, as well as respond to and build on each other's ideas.
- **Support teachers in understanding the value of linking mathematical representations.** The book *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics, 2014) highlights the importance of engaging students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures as tools for problem solving. Linking visual mathematical representations to symbolic and verbal representations is especially helpful for English language learners. Site-based lesson design collaboration or lesson study, using practical CCSS-M implementation resources such as the book *Principles to Actions*, can provide a structure in which teachers can work together to design lessons that support the linking of representations.

Introduction

Implementing the Common Core State Standards–Mathematics incrementally

Over the past several decades there have been numerous calls for improving mathematics instruction, with the goal of improving students' mathematics achievement in the United States. The recent Common Core State Standards-Mathematics (CCSS-M) require teachers to teach in dramatically different, more rigorous ways. Teachers are expected to focus on mathematical reasoning, emphasize multiple approaches for solving problems, and support students to connect various mathematical representations. In addition to content standards, the CCSS-M put forth eight Standards for Mathematical Practice that describe ways in which students should engage with mathematics across grade levels. These Standards include providing opportunities for students to make sense of problems, construct mathematical arguments, and critique the reasoning of others — all of which require teachers to understand what those practices look like and how to help students engage in them. In other words, the CCSS-M require teachers to use common and specialized mathematical knowledge as well as pedagogical content knowledge. This places a heavy burden on teachers not only to understand content at a deeper level, but also to know how to teach it differently than they themselves were taught.

Expecting teachers to successfully implement the major changes called for in the CCSS-M overnight (or even over a couple of years) is unrealistic and does not reflect an understanding of the complexities involved in teaching the content and practices suggested in the CCSS-M. Instead, change may be more likely and sustainable if teachers are encouraged to shift their practice incrementally in a continuous improvement model (Star, 2016; Hiebert & Morris, 2012; Stigler & Hiebert, 2004). Accordingly, instead of expecting teachers to make immediate broad, sweeping changes, the expectation should be for small yet powerful changes that teachers can implement relatively easily in their instruction (Star, 2016). For example, teachers may initially implement manageable new ideas that make sense to them, such as:

- Math talks to support students to conceptualize and represent operations
- Structures and practices to support student-to-student discourse in small-group work
- Counting objects to support students to sort, organize, and count by groups
- Choral counting to engage students in reasoning, predicting, looking for patterns, and justifying things they notice in their counting

Yet, implementing just one or two manageable changes in a teacher's practice is not sufficient for quality mathematics instruction that fully implements CCSS-M content and practice standards. While, for example, incorporating math talks can be a small yet powerful change (Star, 2016), it is the accumulation of these types of incremental changes over time that will most likely result in the ambitious teaching called for in the CCSS-M.

Math in Common classroom observation study

In order to better understand the types of incremental changes taking place in math classrooms as a result of the CCSS-M, WestEd staff visited 141 elementary and middle school classrooms in eight Math in Common (MiC) school districts during the 2015/16, 2016/17, and 2017/18 academic years. Observations were conducted in the spring and fall of each year; some classrooms were visited repeatedly within a year or across years.

Knowledge that math teachers must have in order to teach effectively

Teaching mathematics is complex, demanding work (NCTM, 2014; McDonald, 1992; Jackson, 1968). Researchers have identified and elucidated "knowledge of mathematics for teaching" — the professional knowledge that mathematics teachers must have in order to effectively teach mathematics (e.g., Ball & Bass, 2000; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008). This conception of knowledge of mathematics for teaching is multifaceted and includes both content and pedagogical content knowledge.

The content knowledge that mathematics teachers need comprises both "common" and "specialized" knowledge of mathematics (Ball, Thames, & Phelps, 2008). Common content knowledge is defined as the basic understanding of mathematical skills, procedures, and concepts acquired by any well-educated adult. Specialized knowledge involves a deeper, more nuanced understanding of mathematical skills, procedures, and concepts. It enables teachers to evaluate mathematical representations and solution strategies; analyze (rather than just recognize) errors; give mathematical explanations; and make connections among mathematical strands. It can be characterized as "profound understanding of fundamental mathematics" (Ma, 1999, p. 120).

The pedagogical content knowledge (PCK) that mathematics teachers need includes a sophisticated understanding of effective instructional practices and student thinking related to specific mathematical content, and comes into play during all phases of teaching:

During instructional planning, PCK helps teachers:

- Select curricular materials and sequence content to facilitate student learning
- Predict how their students will approach specific mathematical tasks
- Consider the needs of linguistically and culturally diverse students
- Anticipate student conceptions and typical misconceptions

As teachers conduct lessons, PCK enables them to:

- Recognize instructional affordances and constraints of different representations
- Interpret incomplete student ideas
- Anticipate opportunities to address language or cultural references
- Consider how to respond to various correct or incorrect pathways students explore

After completing a lesson, PCK is central to enabling teachers to:

- Reflect on the learning that did or did not take place
- Consider how to plan for and improve future lessons

Pairs of WestEd researchers observed and analyzed mathematics lessons, documenting the shifts in teachers' instructional approaches and using an observation instrument to rate the quality of the lessons and the instruction. Districts chose the teachers to be observed each year, based on grade level, availability, and interest.

The WestEd team was specifically interested in examining shifts in teachers' instructional practices related to the CCSS-M, and learning whether there were instructional patterns to be found within and across the MiC districts. Accordingly, this report draws on qualitative data — including classroom observations, teacher

Table 1. MiC Observation Instrument categories adapted from the Mathematical Quality of Instruction instrument

OBSERVATION CATEGORY	DESCRIPTION
Teacher work to support richness of the mathematics	
Linking between representations	Teachers' and students' explicit, public (in small or whole group) linking and connections between different representations of a mathematical idea or procedure.
Multiple solution methods or procedures	Multiple solution methods occur or are discussed for a single problem. Multiple procedures for a given problem type occur or are discussed.
Mathematical sense-making	The teacher publicly attends to one or more of the following: the meaning of numbers, understanding relationships between numbers, connections between mathematical ideas or between ideas and representations, giving meaning to mathematical ideas, whether the modeling of and answers to problems make sense.
Student engagement in mathematical practices	
Students provide explanations	Students provide a mathematical explanation for an idea, procedure, or solution. Examples: Students explain why a procedure works, Students explain what an answer means.
Student questioning and mathematical reasoning	Students engage in mathematical thinking that has features of important mathematical practices. There must be clear evidence of students engaging in such practices, such as: Students provide counterclaims in response to a proposed mathematical statement or idea; Students ask mathematically motivated questions requesting explanations (e.g., "Why does this rule work?" "What happens if all the numbers are negative?").

Source: Adapted from Hill (2014).

interviews, and ratings derived from a classroom observation instrument — to present five cases describing shifts in classroom practice related to the CCSS-M. The final WestEd evaluation report, due to be released in February 2019, will provide detailed quantitative analysis of shifts in instructional approaches related to the CCSS-M over three academic years of the MiC Initiative, as measured with the MiC Observation Instrument.

Math in Common observation instrument

In order to systematically observe and rate the mathematics lessons of the MiC classrooms, an observation tool was adapted from two existing classroom observation instruments: the Mathematical Quality of Instruction (MQI) instrument (Hill, 2014) and the Teaching for Robust Understanding (TRU) instrument (Schoenfeld & the Teaching for Robust Understanding Project, 2016). As shown in tables 1 and 2, the adapted instrument focused on eight categories — five from the MQI instrument (Table 1) and three from the TRU

Table 2. MiC Observation Instrument categories adapted from the Teaching for Robust Understanding instrument

OBSERVATION CATEGORY	DESCRIPTION
The mathematics	How accurate, coherent, and well justified is the mathematical content (including mathematical language)? Is there a clear mathematical goal for the lesson? How did mathematical ideas develop within the lesson for students?
Access to mathematical content	To what extent does the teacher support access to the content of the lesson for all students? Who did and didn't participate in the mathematical work of the class, and how?
Agency, ownership, and identity	To what extent are students the source of ideas and discussion of them? How are student contributions framed? What opportunities did students have to explain their own and respond to each other's mathematical ideas? How does the teacher respond to student ideas?

Source: Adapted from Schoenfeld & the Teaching for Robust Understanding Project (2016).

instrument (Table 2). It is important to note that the MQI categories are rated on a 4-point scale (1 = Not Present, 2 = Low, 3 = Mid, 4 = High), while the TRU categories are rated on a 3-point scale (1 = Novice, 2 = Apprentice, 3 = Expert).

A Focused Study: Examining 16 Teachers' Classroom Practice over Two Years

In order to better understand the nature of incremental shifts in teachers' classroom practice as they continued to implement the CCSS-M, WestEd secured permission from four districts to have the same 16 teachers who had been observed twice in 2016/17 be observed again twice in 2017/18, for a total of four observations per teacher over those two academic years. The two charts in this section show the composite MQI and TRU ratings for these 16 teachers across the four observations.

In general, the MQI ratings did not reveal large shifts in implementation of the CCSS-M, as shown in Figure 1.

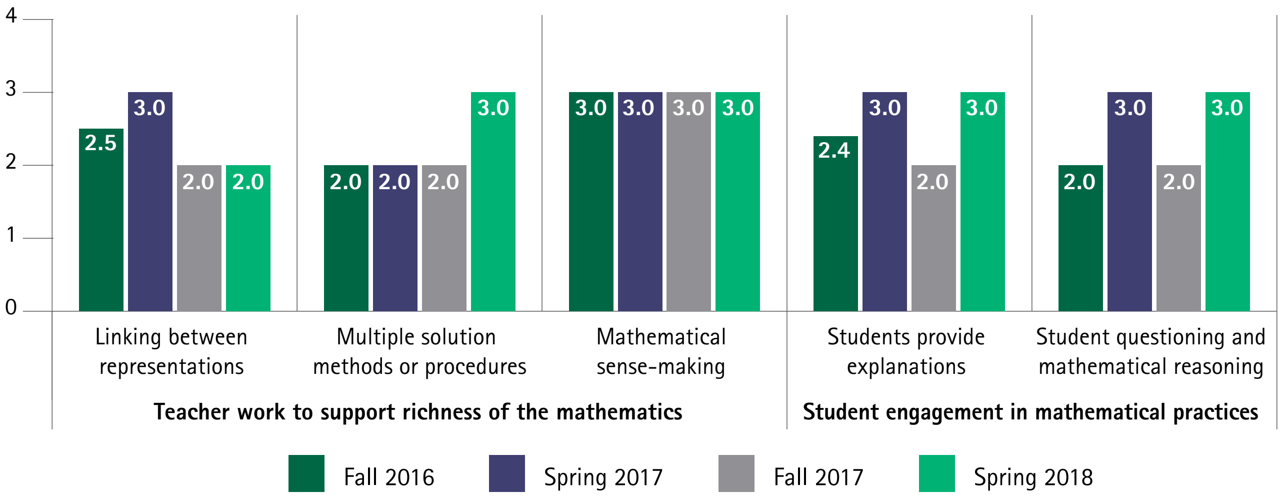
The category of *multiple solution methods or procedures* shifted from an average low rating of 2 to a mid rating of 3. A low rating means that the teacher or student briefly mentions a second method or procedure, but the method is not discussed at length or enacted (for example, the teacher may simply add, "or you could solve it with lattice multiplication"). A mid rating means that multiple solution methods or procedures occur or are discussed (e.g., solving division problems in two ways), but discussion does not explicitly compare methods for efficiency, appropriateness, ease of use, or other advantages and disadvantages.

In the *students provide explanations* and *student questioning and mathematical reasoning* categories, the ratings alternated between low and mid ratings over the course of the four observations. A low score in the *students provide explanations* category means that there are one or two brief student explanations, while a high score means that student explanations characterize much of the lesson. A low score in *student mathematical reasoning* means that one or two instances of brief student mathematical questioning or reasoning are present, whereas a high score means that student mathematical questioning or reasoning characterizes much of the lesson.

Linking between representations was the only category in which the ratings decreased from mid to low. A low rating in the *linking representations* category means that links are present in a pro forma way. For example, the teacher may show a symbolic notation representation such as the fraction ¼ and a rectangle with 1 out of 4 parts shaded, and state that one quarter is one part out of four. These sort of links are not very explicit or detailed. A mid rating means that there is an isolated instance where both representations are visually present and the correspondence between the representations is explicitly pointed out in a way that focuses on meaning.

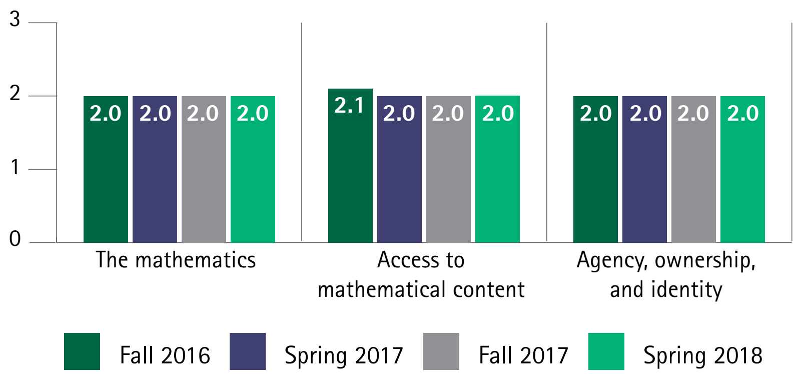
As seen in Figure 2, composite TRU ratings remained consistent across the four observations — the "apprentice" rating of 2. The apprentice level in *the mathematics* category means that while the mathematical content of the lesson is at grade level, the lesson's activities are primarily skills-oriented, with few opportunities for making connections (e.g., between procedures and concepts) or for mathematics coherence. In terms of the *access to mathematical content* category, the apprentice rating means there is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students. An apprentice rating in *agency, ownership, and identity* means that, while students do have a chance to explain some of their thinking, the teacher is the primary driver of conversations and arbiter of correctness; in class discussion, student ideas are not explored or built upon.

Figure 1. Average MQI rating of 16 case teachers, on a scale of 1–4



Note: These ratings are on the following 4-point scale: 1 = Not Present, 2 = Low, 3 = Mid, 4 = High.

Figure 2. Average TRU rating of 16 case teachers, on a scale of 1–3



Note: These ratings are on the following 3-point scale: 1 = Novice, 2 = Apprentice, 3 = Expert.

Key takeaways from classroom observation data

The overall observation ratings of the 16 teachers are fairly typical when examining shifts across two years. There are a few possible explanations for this. It generally takes more than two years to make substantive shifts in classroom practice consistent with the CCSS-M, so the duration of the observation period may not have been long enough to reflect significant shifts in teachers' practice. In addition, the observation instrument may not be fine-grained enough to pick up any incremental shifts in practice that did occur during the two years of observation. Also, change can happen unevenly across lessons. In order to support teachers to continually improve their CCSS-M–aligned instruction, it might be helpful for districts to look at the categories in which there is uneven growth — such as *linking between representations*, *students providing explanations*, and *student mathematical reasoning* — and provide targeted professional development for teachers in those areas.

Five Cases of Shifts in Classroom Practice

While the data displayed in Figures 1 and 2 show little to no change in ratings over the two years, observers' field notes and interviews described evidence of small, sometimes subtle, shifts in practice that, while not sufficient to move the rating up to the next level, showed progress in teachers' implementation of the CCSS-M. After analyzing the 16 teacher ratings, observation notes, and interviews across the two years, five themes emerged in which teachers exhibited common shifts in their classroom practice. To elucidate these themes, we provide cases that contextualize the teachers' work and their incremental shifts in instruction. The themes of the first three cases are directly related to categories from the MQI/TRU observation protocol:

- **Representing and linking mathematical ideas.** This dimension of the MQI looks for evidence of the opportunities that teachers provide for students to engage in rich mathematical reasoning and doing mathematics — specifically, how mathematical ideas are represented and connected to support student understanding.
- **Access to mathematical content.** The TRU framework highlights access to mathematical content, asking the following questions: *To what extent does the teacher support access to the content of the lesson for all students? Who did/didn't participate in the mathematical work of the class, and how?*
- **The mathematics.** The TRU framework focuses on the accuracy and coherence of the mathematical content of a lesson, asking the following questions: *Is there a clear mathematical goal for the lesson? How did mathematical ideas develop within the lesson for students?*

The themes of the final two cases emerged from observation notes and teacher interviews over the course of the two years:

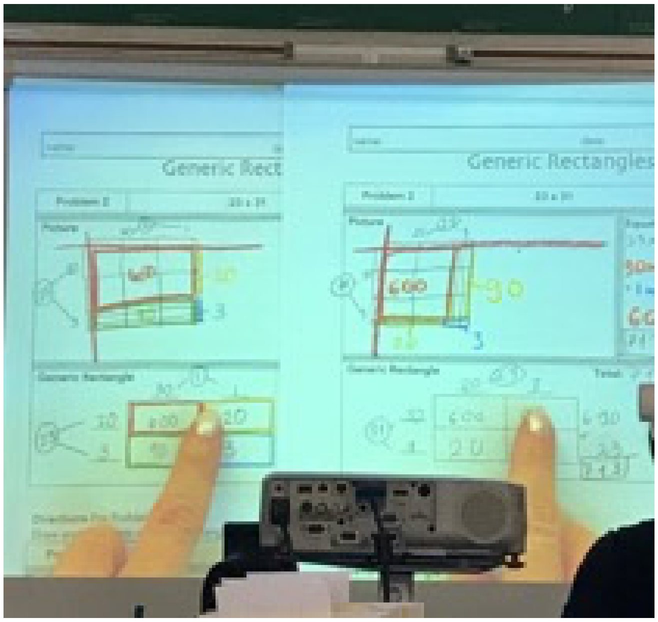
- **Collaboration supports teacher identity.** While the TRU framework talks about *agency, ownership, and identity* in terms of students as learners of mathematics and highlights attributes of students' identities as doers of mathematics, in this case, we consider teachers and the changes in practice they make as they develop robust identities as teachers of mathematics.
- **Continuing to learn and change.** Two veteran teachers, active participants in professional learning opportunities in their districts and at their sites, implement new pedagogical strategies in their classrooms to help students use the Standards for Mathematical Practice as they learn and do mathematics.

Case I: Representing and linking mathematical ideas

Over the course of two years, we conducted four classroom observations of Jackie and Mary, two 4th grade teachers from the same school. During each observation, the two teachers taught the same lesson, which they had planned together, each incorporating their own personal styles. Both teachers commented that as they deepened their understanding of and comfort with the math content, they were better able to help students represent and link mathematical ideas in multiple ways and recognize the connections among the representations.

In talking about her mathematics teaching, Jackie reflected a lot on her own experience as a mathematics

Figure 3. Using color across representations of the distributive property



Source: Photo taken by WestEd during fall 2017 lesson observation.

learner, remembering that she needed opportunities to process that were not available to her, and she sees the value of that for her students. She indicated that she appreciates that the CCSS-M focus on having students understand concepts and not just learn procedures. She stated that getting involved in her district's discourse team and working with other teachers to learn how to implement conceptual lessons has enhanced her classroom practice. She also noted that she has learned to appreciate the importance of productive struggle.

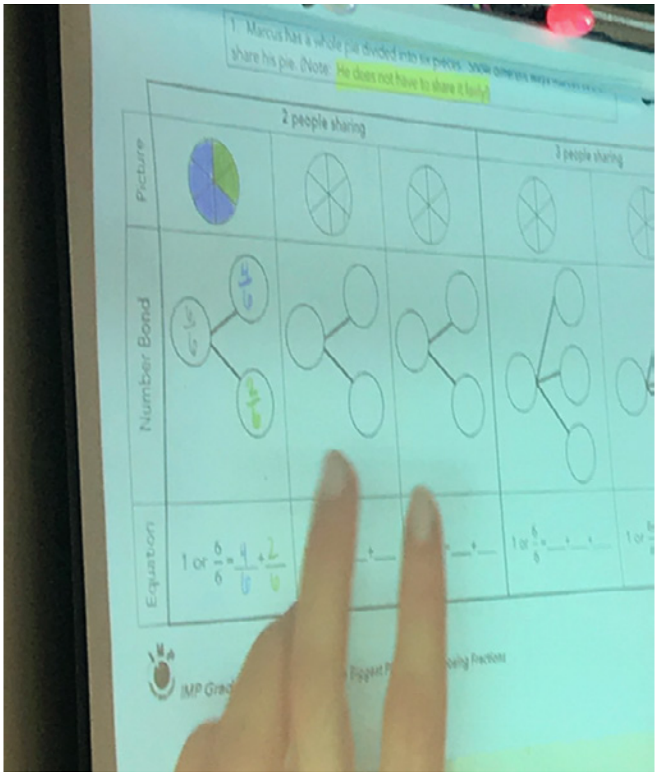
Mary had been fearful of teaching math because she only knew how she had been taught, and she knew that was not going to meet her students' needs. As she moved forward with CCSS-M implementation, she knew she wanted her students to understand mathematics, not just be able to "do it." She has given herself permission to pay attention to students and to make choices and adjustments in the moment as needed. She credited her involvement with the district's discourse team as making a big difference in her teaching. Having been invited by a colleague to join the group, she has benefitted from working together with other teachers in workshops and summer math institutes to co-plan and co-teach, trying out new lessons and strategies.

In fall 2016, we observed lessons on multiplication by powers of 10 in which both teachers had some struggles with the lesson but showed great perseverance in trying to support students. Representations seen included base-10 blocks and a place-value chart. In Jackie's class, she presented the linking between these representations quickly at one point during the lesson when students were having difficulty with place value. In Mary's class, she showed the same representations, but for the most part, she demonstrated and had students copy what she had done, including use of some color coding. Because Mary was as yet unable to link among the representations, observers assigned a "not present" rating for the *linking between representations* category.

One year later (fall 2017), we observed lessons on two-digit multiplication using the distributive property and a generic rectangle to represent it (see Figure 3). The lesson activities supported students in transitioning from drawing "literal" area models to using a generic rectangle to represent the distributive property. In both classes, we observed the teachers explicitly and consistently using color to show how factors and products match up across representations — area model, number sentence, and generic rectangle. Attending to Standard for Mathematical Practice #7: *Look for and make use of structure*, both teachers demonstrated consistency and attention to detail in their own recording of linking representations, and they encouraged and supported students to do the same.

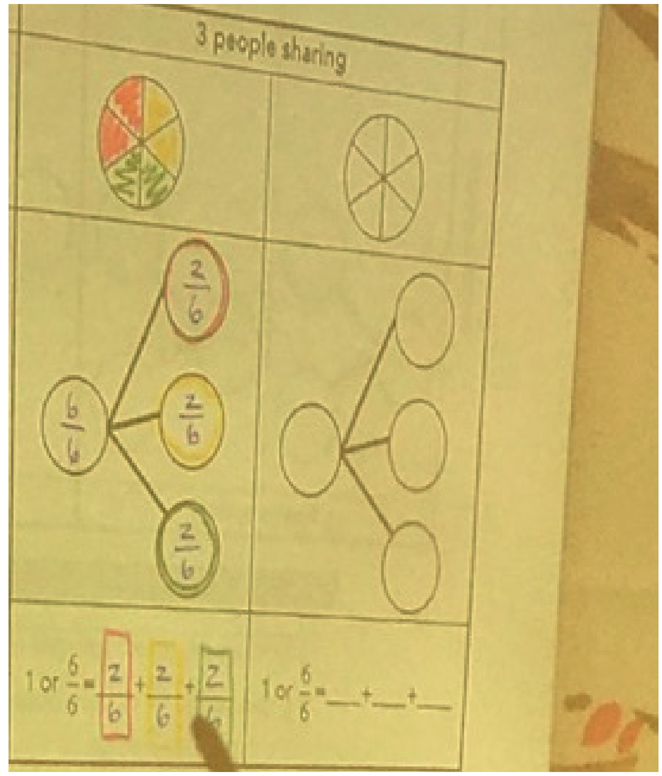
In spring 2018, the observed lessons focused on fractional shares. Representations included drawings, number bonds, and number sentences (see Figure 4). Both teachers explicitly used color to show connections across the three representations. As Jackie recorded, she was explicit with students: "I make my colors match because it helps to keep track." In Mary's class, she highlighted a student's sharing of how he had shown these connections. It was clear from observing students

Figure 4. Linking drawings, number bonds, and number sentences



Source: Photo taken by WestEd during spring 2018 lesson observation.

Figure 5. Using color coding to highlight mathematics



Source: Photo taken by WestEd during spring 2018 lesson observation.

working in both classes that they have had many opportunities to work on linking among representations, and they valued the practice of color coding to help them do so. In both classes, we saw examples of the teachers providing feedback and suggestions to help students make better use of the color coding. For example, when one group used a dark color that ended up hiding the numerical values, the teacher suggested either just outlining or using a lighter color (see Figure 5).

Conclusion

Both of these teachers showed incremental growth in their mathematics teaching over the course of the classroom observations. Working together in planning and reflecting on their lessons has helped them refine their own thinking and consider ways to help students do the same. Jackie noted that she has learned to appreciate the importance of productive struggle and supporting students with tools such as color coding to help highlight mathematical ideas. Mary agreed, saying, "I feel like the whole process [professional development opportunities and collaborative planning] has strengthened who I am as a math teacher. It's strengthened my students' understanding, and I feel like they're better prepared to go forward. And I'm very, very grateful for that experience." She added proudly that she now has more students on the honor roll because of better math performance — math had been the subject that was holding them back.

Case II: Access to mathematical content

The TRU framework highlights access to mathematical content, asking the following questions: *To what extent does the teacher support access to the content of the lesson for all students? Who did/didn't participate in the mathematical work of the class, and how?*

Observations of middle school teacher Tanesha and 4th grade teacher Deidre three times over two years provided evidence of shifts in each teacher's access practices from apprentice (rating of 2) to expert level (rating of 3). The apprentice level is defined as "uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students." At the expert level, "a teacher actively supports and to some degree achieves broad and meaningful mathematical participation or what appear to be established participation structures that result in such engagement."

Middle school teacher Tanesha

Tanesha teaches at an urban middle school that traditionally has the lowest test scores in the district. The student population is 84 percent Hispanic, 10 percent Black, 4 percent Asian, 1 percent White, and 1 percent other, with 92 percent of the students eligible for free or reduced-price lunch. In her interview, she reflected about how far she has come as a teacher over the past five years. She said that in the past, she taught using "direct instruction" and now she "allows students to explore concepts based on their prior knowledge, use a variety of strategies, and listen to and build on each other's ideas." She attributes this change to the supportive structures she has — her principal, who "lets them take risks and trusts us," and her department chair, who collaborates in lesson planning and brings her interesting tasks, lessons, and projects. In addition, she talked about her district mathematics instructional team, which provides her with valuable professional learning opportunities both within the district and outside the district at mathematics conferences.

In our first few observations of Tanesha, we saw varied student access opportunities, such as collaborative groups/pairs, oral reading, manipulatives, and student questioning. While there appeared to be uneven access or participation, it was evident that Tanesha made efforts to provide mathematical access to her students. While her students had some opportunities to share ideas and methods with groupmates, agree or disagree on mathematical approaches and methods, ask questions, and make comments about a student idea, Tanesha remained the primary arbiter of correctness, and student explanations were mostly procedural in nature.

At the final observation in spring 2018, students were provided ample opportunities to be the source of ideas and discuss them with each other in small groups. The lesson flowed smoothly and the students' comfortable use of the participation structures and engagement in the lesson's activities suggested that this lesson was typical of their daily math class. Students explained their reasoning, argued their mathematical logic, and responded to each other's ideas with respect and substance. When students shared their thinking, Tanesha asked other students to examine the thinking and talk with their partners. For example, she prompted them with questions such as "What is he thinking? Why is she doing what she is doing? What is his reasoning?" Tanesha actively supported, and for the most part achieved, broad and meaningful participation in mathematics for all of her students. She had established participation structures that supported student engagement, such as individual think time, partner or small-group work, additional time when needed, and being allowed to ask a friend when students were stuck during whole-class discussions. In addition, Tanesha made several efforts to connect her students to the mathematical context in various ways. For instance, she adapted the lesson's task to use her students' names and the school's name, and the homework assignment to plan a field trip was about an actual field trip they would be taking soon. Students were also given the autonomy to determine their own real world context for their "Deal or No Deal" extended project, helping them to see the usefulness of the mathematics they are learning in their own lives.

Fourth grade teacher Deidre

Deidre teaches 4th grade in a suburban, non-Title I elementary school. The student population is 32 percent Asian, 18 percent Hispanic, 15 percent Black, 10 percent Filipino, 7 percent White, and 18 percent other, with 62 percent of the students qualifying for free or reduced-price lunch. In her interview, Deidre reflected on the past five years and how her teaching practice has changed. She said that conceptual understanding has come into focus now — that procedures alone do not work. She said that she "now focus[es] on supporting students to solve problems in different ways, represent their ideas visually, and defend their solutions." She attributes her learning to the "great PD" she has received over the past five years from her district and the access she has had to great speakers outside of the district, which has led to continual conversations with her colleagues about implications for her classroom practice.

In our first observations of Deidre, she called on students using a random selection by pulling a stick with their name on it. Although she encouraged students to speak out to their fellow classmates, students often spoke so quietly that their peers could not hear them. While students were given the opportunity to come up with original mathematical thinking and encouraged to work together, they did not discuss their solutions with each other. Deidre was the arbiter of mathematical correctness.

By the final observation in the spring of 2018, students were clearly used to engaging in mathematical discourse in ways not observed previously. They articulated their thinking to each other, asked each other mathematical questions, and showed respect for each other's ideas. All students appeared to be engaged and involved in the mathematical task as they solved and discussed problems. There were multiple instances throughout the lesson of students making sense of the mathematics as well as commenting on, questioning, and respecting others' ideas. The students were clearly used to engaging in mathematical discourse and articulating their ideas to each other. Deidre appeared to be attending to all voices in the discussion. She was engaged as a partner in the discussion, nudging as needed but remaining open to having students build on each other's thinking.

Conclusion

Even though Tanesha and Deidre showed incremental shifts of just one rating point in access to mathematical content, their shift from a rating of 2 to 3 represented a move from "apprentice" to "expert." Both teachers' initial observations showed mixed opportunities for students to access and own the mathematical content in the lessons. By the final observation, while the shift in observation rating score was incremental, the shift in classroom culture was dramatic. All students appeared engaged as they solved and explained their thinking, asked each other questions, and built on each other's ideas and methods. Both Tanesha and Deidre attended to all voices in the discussion. Students' opportunities for learning mathematics had increased in important ways.

Case III: The mathematics

The Common Core State Standards require teachers to substantially improve the rigor and coherence of their lessons in ways in which most of them have not experienced as learners themselves. As stated earlier, these instructional shifts require teachers to have flexible procedural and conceptual mathematical knowledge. The findings from two years of observations of teachers Mariah and Marcus highlight the importance of supporting teachers' pedagogical content knowledge.

Sixth grade teacher Mariah

Mariah had a great deal of mathematical support in her journey to implement the CCSS-M. In her interview, she reflected on her past teaching: "'Do this, now practice it, show it on whiteboards, now you've got it, let's move on!'" She compares that to her current teaching, in which she focuses on students' "productive struggle and mathematical explanation and reasoning, and on encourag[ing] them to think about the strategies they are using." She talked about how her questions have become more purposeful and focused on what students are thinking mathematically and recognizing multiple approaches and strategies that work, rather than just a single "right" way ("the way we were taught").

Mariah teaches 6th grade in an urban district middle school. The student population is 69 percent Hispanic, 15 percent Black, 9 percent Asian, 4 percent White, and 3 percent other, with 87 percent of students qualifying for free or reduced-price lunch. In her first observation in the fall of 2016, Mariah's lesson focused on having students work in groups to translate word problems into numerical expressions, then algebraic ones. She then asked students how they decided which operation to use and which order to put the numbers in. For the most part, Mariah emphasized locating the "key words." When Mariah asked, "How did you know you needed to multiply? How did you know you needed to divide?" students mostly answered, "Because it says 'per hour'" or "Because it says 'for each.'" The precise mathematical goal of the lesson was not clear, and it was difficult to determine what students had accomplished by the end of class, even though the lesson was reasonably accurate and somewhat coherent.

Across the next three observations, Mariah's lessons continually improved. By the spring of 2018, Mariah received an expert (3) rating in *the mathematics* category of the TRU framework. The entire lesson focused on making sense of two-step equations through a coherent set of activities (using a Mathematics Assessment Project Formative Assessment Lesson[1]). Throughout the whole-group and small-group work, Mariah asked questions such as "What was your thinking?" "How did she convince you?" and "How do you know…?" Mariah made it clear that her expectation was for her students to communicate their reasoning, not simply to find the answers.

Mariah credits her improved teaching to the opportunities she has had in her school and district to collaborate with colleagues on unit studies, lesson studies, and grade-alike meetings across school sites. In addition, she has a district-level math coach who regularly observes her classroom and provides her with content-specific feedback on her lessons and her teaching moves and strategies. Mariah volunteered her classroom and students for district lesson study focused on high-level cognitive demand tasks and the design and delivery of five practices for orchestrating productive math discussions. In addition, Mariah has a supportive principal (a former mathematics teacher) and a department chair who collaborates with the math department in planning lessons.

Fifth grade teacher Marcus

Marcus teaches 5th grade in a school whose student population is 100 percent Hispanic, 89 percent eligible to receive free or reduced-price lunch, and 66 percent English learners. In all four observations, the classroom learning environment was pleasant and comfortable. Marcus was very caring and had a wonderful rapport with his students. Yet, across the four observations, there was no evidence of shifts in his teaching practice. Marcus was trying out new strategies, implementing number talks and using new mathematical tasks such as problems of the month, yet there were not real opportunities for students to make sense or generate different strategies. Even in mental math activities, students shared that they were "doing the algorithm in the air" when they solved the problem. Each lesson was more procedural than conceptual in nature — primarily involving Marcus listing steps and reading solutions, rather

1 http://map.mathshell.org/lessons.php

than asking students why solutions worked or made sense. Students presented posters of their group work on the problem of the month (a primary-mathematics-level problem) and it was clear that they had been working on how to give each other feedback (which they did in a very respectful way). However, the feedback was almost totally about presentation style — "I like the way you spoke clearly" or "You did a good job of pointing to what you're talking about" — and very little about the mathematics or the problem-solving approach.

Marcus came to elementary school after teaching secondary geography, and he indicated that he thinks a lot about how he can connect across disciplines and engage his students. In the interview, Marcus said that his teaching for the most part has not changed over the past several years, but that he is using more charts (posters) and trying to "slow the process down" for his students. He says that he is focusing on communication and writing in mathematics and asking students to explain their thoughts more now. His reflections highlight his attention to communication of ideas in general, but he does not seem to have had the opportunity to learn the pedagogical content knowledge that includes a sophisticated understanding of effective instructional practices and student thinking related to specific 5th grade mathematical content.

Conclusion

Both Mariah and Marcus are caring, thoughtful, and dedicated teachers. Yet each teacher had a different level of pedagogical content knowledge and mathematical support in their journeys to implement the CCSS-M. Mariah had a great deal of district and school-site mathematical support, in the form of lesson study, collaborative planning, and content coaching, that allowed her to gain a deeper, more nuanced understanding of the mathematical skills, procedures, and concepts her 6th grade students needed. Marcus, on the other hand, mostly worked independently to improve his practice and implement the CCSS-M, without the opportunity to gain the mathematical knowledge needed for teaching his 5th graders in a standards-aligned way. Mariah reflected that her teaching over the past five years is "completely different," shifting from a procedural focus to a conceptual focus, in her view. Marcus was trying new pedagogical strategies — such as number talks, problems of month, new tasks, and student communication — that focused more on presentation style and mathematical procedures than on making connections across mathematical strands. If Marcus had the opportunity to experience the level of pedagogical content support that Mariah has had over the past five years, it is likely that his practice would have shown more improvement.

Case IV: Collaboration supports teachers' identity

The TRU framework talks about "agency, ownership, and identity" in terms of students as learners of mathematics and highlights attributes of students' identities as doers of mathematics. The notion of identity is also a helpful way to think about teachers and the changes in practice they make as they develop robust identities as teachers of mathematics. Observations of teachers Connie and Elizabeth over two years provide evidence of small, but important, changes that both teachers have made in their mathematics instruction and in their identities as mathematics teachers.

While neither Connie nor Elizabeth showed significant growth on the MQI or TRU rating scales, in both cases, their mathematics lessons had an updated look by spring 2018. This highlights the fact that changes in practice are not always reflected in changes in rubric ratings. Both teachers talked about how, through collaboration with colleagues and participation in district-sponsored professional learning opportunities, they have come to see themselves as teachers of mathematics.

Third grade teacher Elizabeth

Elizabeth is a third grade teacher, teaching math in a self-contained classroom. In describing her growth as a math teacher, Elizabeth reflected on having learned the value of discourse to support student understanding. She says her classroom is a lot noisier now than it was several years ago, and she sees that as a good thing. Elizabeth belongs to the discourse team in her district and credits that collaboration with helping her get better at engaging students in mathematical talk. She commented, "Before I was just [saying to students], 'Well, tell your neighbor your answer,' and in my head, that was talking. I wasn't really asking a deeper question or giving them more time to discuss." The discourse team helped Elizabeth understand the importance of engaging students in mathematical discourse (for example, so they can "construct viable arguments and critique the reasoning of others" as described in Standard for Mathematical Practice #3) and provided her with opportunities to co-plan tasks, share student work, and refine her practice.

A good example of Elizabeth's shifting practice is seen in her implementation of problem-solving lessons. When we first visited in fall 2016, she introduced the lesson with a "Read 2 Ways" strategy, intended to help provide all students access to the task. However, due to her very procedural implementation of the strategy, a number of students still had difficulty engaging with the task. In our second observation, we saw Elizabeth again attending to access for students, this time by selecting some pictures to help students understand a fence as the perimeter of an area. However, the pictures she chose actually created some distraction for the students, with some of the students focusing more on drawing a fence than on how much fencing would be needed.

Our 2017/18 observations showed evidence that Elizabeth's perseverance and continued participation in professional learning opportunities were paying off. In the fall, it was great to see a lesson launch that provided students access to the problem through pictures and through discussion about the problem stem before beginning to work with specific numbers to answer the lesson's question. In the spring, Elizabeth's lesson used the same garden/fencing problem as in the previous school year, but with improved implementation, leading to greater student success with the task.

Sixth grade teacher Connie

Connie is a sixth grade teacher, teaching math in a self-contained classroom. She came to sixth grade from third several years ago, and she shared that when she first made the shift, she felt challenged in teaching math well. She was not sure that her content knowledge was sufficient to meet the demands of the sixth grade standards. She described her teaching as very textbook-driven and procedural at first. She said that she used to think that she "had to spoon-feed them everything. They don't know. They can't figure that out. It's too hard."

Taking advantage of district-offered professional development and engaging with teachers at her site through their professional learning community (PLC) have made a difference in Connie's teaching, and in her students' experiences of mathematics as learners. She credited the professional development offered through her district with deepening her content understanding, allowing her to better help students develop conceptual understanding. She also highlighted the importance of her site-based PLC and noted that it has become a real learning community focused on analyzing student work and using evidence to inform instruction. She said, "Now it's just second nature to go to your PLC meeting and bring your data, and [based on weaknesses that the student work shows, say] 'Let's adjust here'…" We first visited Connie's class in fall 2016, when she was just beginning to let go of her math textbook as the primary driver of her instruction. In that lesson, we observed a warm-up brain teaser, a few mental math problems, then a lesson on exponents, with about 60 percent of the class time focused on textbook problems — first modeled by the teacher, then practiced by the students. In spring 2017, the lesson we observed included a warm-up, some mental math, modeling operations with algebra tiles, and a problem-solving task. The textbook had a less prominent role, and each segment of the lesson was internally coherent with grade-level-appropriate content. However, the full lesson lacked coherence in terms of the four segments making a "whole" mathematical idea.

In the third and fourth observations, we saw additional efforts to encourage student sense-making. Both lessons began with accessible openers that engaged all students ("Which One Doesn't Belong?" and "Estimation 180") and number talks, then shifted to the content focus for the day. While the openers provided students with opportunities to engage in the mathematical practices as they solved problems, the content-focused segments of the lesson were over-scaffolded, taking away some thinking opportunities. That said, students were more engaged in thinking and problem-solving opportunities before going to the textbook for practice than we had seen in previous lessons. Connie was continuing to try new ideas and was still learning to connect the various activities and ideas into a coherent whole — which all showed evidence of her ongoing incremental change.

Conclusion

We saw some changes in instructional practice in both classrooms over the two years, and there is still progress to be made. Substantial change to one's instructional practice takes time and effort. A recurring theme throughout the interviews with both Elizabeth and Connie was the important role of professional learning and interaction with colleagues. Both teachers have come to see the textbook more as a resource to support learning than as the driver of lesson planning, and they value their collegial discussions for helping them make strategic choices. They both also talk about feeling more confident in their teaching of math — reflecting the growth in their identities as math teachers — while at the same time recognizing that they have room to grow.

Case V: Continuing to learn and change

Veronica and Rachel were both preparing for retirement after long teaching careers when we made our final visits to their classrooms. It was inspirational to see that both of them were still full of energy and love of teaching math.

In interviews, both teachers talked about the evolution of their math teaching over the years. They shared examples of the ongoing incremental changes they have made and their reasons for doing so. As veteran teachers, both had taught math very traditionally, in the way they had been taught, for many years. They did not know any differently. As Veronica put it, "I was very isolated. Just turned to the next page whether you liked it or not — you just did what it said." Both she and Rachel commented that as they learned more about the CCSS-M through their district and site-based professional learning opportunities, they became increasingly aware of the need to do something different in their mathematics teaching. And the payoff, as Rachel put it, is that "the kids are doing more of the thinking. I'm doing the groundwork, but kids are doing more of the heavy lifting intellectually."

First grade teacher Veronica

Veronica teaches first grade in a suburban school district. While she stated that she had taught pretty traditionally most of her career, she also recognized a number of years ago that there was room for something different, so she started to add some math games into her instruction. In doing so, she realized that she needed support and tools for her lessons to be more effective. She credits the district's professional learning efforts — both district-based workshops and site-based PLCs — with giving her those tools. One key learning that Veronica noted, related to the adopted instructional materials, was, "They [district leaders] really taught us how to look at those materials critically." She learned that she did not need to assign every problem on every page, but, rather, needed to consider which activities would support students learning the math content. As a result, she began incorporating Three-Act Tasks and using manipulatives more often, using the textbook to augment these kinds of activities.

In one lesson, Veronica engaged her students in a "counting collections" activity as an opportunity for her to informally assess their progress in recognizing and grouping into 10s. She assigned collections to pairs of students (appropriately sized to differentiate for student needs), and they had the option to choose appropriate tools, such as cups, 10-frames, and hundreds charts. After groups had counted their collections, Veronica strategically chose pairs to share their strategies. The students that shared included:

- A pair that sorted by color and could not figure out how to use that to help them count them all
- A pair that started with grouping by 5, then decided to put pairs of cups with 5 each on top of each other so it would be easier to count by 10s
- A pair that counted by 10s (using cups) to get 11 cups plus 9 more

As each pair shared, there was a brief discussion of what worked and what were some challenges. After asking the last group how many items they had altogether, the teacher then followed up with, "And how many more would it take to make 120?"

Third grade teacher Rachel

Rachel teaches third grade in an urban elementary school with a very diverse student population, many of whom receive free or reduced-price lunch. In our final interview with her, Rachel shared that a huge learning for her was to not be "afraid to have the kids do really challenging things." She talked about having recently visited a school in her district, with students from predominantly higher–socio-economic-status households, and realizing that "my kids are equally bright; they're equally capable" as the others. That realization confirmed for her that the instructional shifts she has made, in which "kids are doing more of the heavy lifting intellectually," were the right thing to do. To this end, Rachel has incorporated number talks and problem solving on a regular basis.

It was interesting for observers to notice the development of Rachel's use of number talks over time. In the first year of our observations, Rachel's number talks were rather simple and short. In fall 2016, she used a contextual problem naming four students in the class who each have four of the school's "bonus dollars," posing the question: "How many [school name] dollars do these students have altogether?" Beyond having a variety of students share their answers, there was little focus on the strategies they used and how those strategies might be useful in other computations. In spring 2017, we observed a "naked numbers" talk with two multiplication problems: 12 x 2 and 12 x 4. In both of these problems, students not only got computation practice, but also learned a variety of strategies (such as repeated addition and doubling addends or factors) that could support their computational fluency.

During the year two visits, both of the number talks observed contained strings of related problems, rather than individual, unrelated computations. In the fall, the problems were 3 x 10, 3 x 20, and 3 x 19. After each was presented, there was a quick sharing of solutions and strategies, with no discussion after the three to highlight how the problems could be related. In the spring, there were again three problems: 3 x 20, 3 x 4, and 72 ÷ 3. Again, each problem was discussed for solutions and strategies, and this time Rachel added a step designed to strengthen students' computational fluency, asking, "What do all three of these problems have in common?" Students responded by noticing the common factor of three and that the products of 60 and 12 added up to 72. One student then concluded that since 60 + 12 = 72, you could add the factors of 20 and 4 to get 24 for the quotient of 72 ÷ 3.

Conclusion

These two veteran teachers definitely did not have short-timers' mentalities. They continued to be active participants in professional learning opportunities in their districts and at their sites up until the end of their teaching careers. Their development in the use of new pedagogical strategies in their classrooms illustrate that small changes can make a big difference, helping students use the mathematical practices as they learn and do mathematics. Having now retired from the classroom, both of these teachers are excited to continue to share what they have learned with other teachers. They have each agreed to help out in their districts beyond their retirements. As Rachel put it, "I should be helping to pass the torch so that other people can do this."

Key takeaways from classroom observation cases

Implementation of the Common Core State Standards for Mathematics (CCSS-M) asks a lot of teachers. It requires them to shift their practice to focus on representing and linking mathematical representations, supporting students to explain and reason about their thinking, preparing for and using a variety of methods and procedures, and deploying flexible procedural and conceptual mathematical knowledge. Implementation of these standards may improve both instruction and student achievement (Schmidt & Houang, 2012). Yet past efforts suggest that instead of expecting broad-scale immediate changes in classroom practice, small yet powerful incremental improvements may result in lasting and continuous instructional progress (Star, 2016).

The five classroom observation cases in this report illustrate the varied ways in which teachers work to incrementally implement the CCSS-M. Jackie and Mary collaboratively planned lessons to focus on the purposeful use of color to connect mathematical representations. Tanesha and Deidre displayed shifts in supporting students' access to the mathematical content of the lessons and ownership of the ideas. Mariah and Marcus illustrate the importance of supporting teachers' pedagogical content knowledge. Connie and Elizabeth became more confident in their own mathematics teaching and their identities as mathematics teachers, trying out new ideas such as number talks, which resulted in increased opportunities for their students to make sense and problem solve. Finally, Rachel's and Veronica's continuous incremental shifts in practice resulted in increased use and improved implementation of math activities such as number talks and counting collections.

Recommendations for Supporting Teachers' Implementation of the Common Core State Standards–Mathematics

Based on our focused study of classroom practice over two years in MiC, we offer the following set of recommendations to other districts who seek to build support for teachers as they work to implement the Common Core State Standards-Mathematics (CCSS-M).

- **Support teachers in making continuous incremental changes.** All of the cases illustrate the importance of honoring teachers as they dip their toes into implementing the CCSS-M. Small but powerful changes that teachers can implement fairly easily in their practice can allow teachers to try out new ideas without feeling as though they need to totally overhaul their teaching. These changes can be viewed as modifications to what they are already doing, thereby increasing the likelihood of sticking. Examples of small changes include number talks, counting collections, problems of the week, choral counting, or small-group student discourse.
- **Develop structures and expertise to support increased mathematical content knowledge.** As the Mariah and Marcus case highlighted, teachers who have multiple levels of support for their pedagogical content knowledge gain more flexible procedural and conceptual mathematics knowledge that helps them implement the CCSS-M. Types of support that can help teachers continuously improve their mathematics teaching include site-based lesson design collaboration, knowledgeable principal support, content-focused professional development, content coaching and mentoring, lesson study, and cross-district grade-level meetings.
- **Support teachers in providing all students with access to mathematical content.** In order for teachers to implement the CCSS-M effectively for all students, classroom activity structures should invite and support the active mathematical engagement of all of the students in the classroom in a meaningful way. Classrooms in which a small proportion of students get most of the attention are not equitable, no matter how rich the content. The TRU framework indicates that equitable mathematics requires a teacher to support *all* students' access to the content of the lessons. Teachers need support to understand how to actively support broad and meaningful mathematical participation, and how and when to use successful participation structures. In addition, teachers need to understand what it means in practice for students to actively share and discuss their ideas. Focusing on discourse strategies can help students learn to explain their ideas and reasoning, as well as respond to and build on each other's ideas.
- **Support teachers in understanding how to link mathematical representations.** The book *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics, 2014) highlights the importance of engaging students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures as tools for problem solving. Linking visual mathematical representations to symbolic and verbal representations is especially helpful for English language learners. In their classrooms, Jackie and Mary worked collaboratively to link representations through the purposeful use of

color. Site-based lesson design collaboration or lesson study can provide a structure in which teachers can work together to design lessons that support the linking of representations.

» **Utilize research-based, classroom-focused resources such as the book *Principles to Actions: Ensuring Mathematical Success for All.*** Several MiC districts have used this book to support principals' understanding of what it looks like to implement the CCSS-M in classrooms. Other MiC districts have used it with teachers in a book study. Still others have used it to support designing and implementing lesson-study lessons. The book has proven to be a practical guide for many in their support of teachers as they work to implement the CCSS-M.

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